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TO THE DEVELOPMENT OF THE KOLMOGOROV K-62 THEORY UNDER THE CONDITIONS OF INTERMITTENCY OF DISSIPATIVE FLUID

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Abstract

Current problems of statistical modeling of the small-scale turbulence structure in the hydrodynamically inhomogeneous turbulent flows are revealed and new concepts are developed.At that the focus is on analyzing and solving the problem, connected with taking into account the effects of internal intermittency of dissipative fluid. The conceptual issues of that statistical modeling, called as the *ASMTurbS* theory, are provided, and the justification of the lognormal law of the distribution of values of the partially averaged energy dissipation in the dissipative fluid of turbulent flow is given. It is shown that the Kolmogorov *K-62* theory is the foundation on which it is possible the further effective constructing of the theory of small-scale turbulence.

1. Introduction

The problem of effective consideration of hydrodynamic intermittency in statistical modeling of developed turbulent flows is well known. Especially acute this problem is shown in the modeling of the small-scale turbulence structure, developed by Kolmogorov and known as the theory *K-62* [[1]].It turns out that in the *inertial interval of scales* the assumption of local isotropy of the small-scale turbulence is not generally satisfied; the hypothesis of the lognormal distribution law of values of the partially averaged dissipation of turbulent energy didn't find its convincing confirmation;the Kolmogorov universal constants C_k, $C_\varepsilon$and μ,included in the expression of the structure functions$S^{(n)}(r) = \langle v^n(r)\rangle \sim r^{\zeta(n)}$with a "scaling" index$\zeta(n)$for the *n*-th moment of the velocity difference$v(r) = \big(\boldsymbol{u}(\boldsymbol{x}+\boldsymbol{r}) - \boldsymbol{u}(\boldsymbol{x})\big)\cdot \boldsymbol{r}/r$,$r = |\boldsymbol{r}|$and their spectra, are not such of that.

This conclusion follows from numerous theoretical and experimental data [[2], [3], [4], [5]], presented also in[[6],[7], [8], [9], [10],[11] , [12], [13]]. In this regard have been developed various models (for example, β*-model* and*log-Poisson model*, see [8]) with an attempt to eliminate these disadvantages.However, in all these models it was assumed that the small-scale turbulence structure in the *inertial interval* is homogeneous (more precisely, locally isotropic), while the constants included in the expressions for the structure functions $S^{(n)}$ were considered universal.At the same time the search for the Kolmogorov universal constants was not crowned with success.So, for example, a huge number of conducted experiments gave different values of the coefficient μin the variation range $0.14 < \mu < 0.5$, [3, 6].Moreover, the results of experimental studies [3] convincingly showed the dependence of this coefficient on the value of the external intermittency coefficient γ.

The purpose of our paper is to show that Kolmogorov's theory specifically is the foundation on which it is possible to construct a theory of the small-scale turbulence that is free of these shortcomings. In particular, we show that the lognormal law of the probability distribution of the partially averaged energy dissipation ε_l is valid only for the dissipative fluid of turbulent flow, and that only for this fluid the coefficient$\mu$is a universal constant, whereas for the turbulent fluidrepresents a statistical quantity that depends on the magnitude of the coefficient γ.

2. The main provisions of the ASMTurbS theory

Autonomy of modeling of the hydrodynamic fields of a dissipative fluid [2]leads to a theory with the ideology of the *ASMTurb* method, [[14]].Therefore, our theory of small-scale turbulence is logical to call the theory

ASMTurbS, i.e. the theory of autonomous statistical modeling of small-scale turbulence. Let us give the main points of this theory.

In [2] was introduced the concept of a dissipative fluid (with the index d) of a turbulent flow. It is recognized that any turbulent flow contains space-time regions with a turbulent G_t and non-turbulent G_n fluid, that is, the generalized domain of such a flow $G = G_t + G_n$. At that, at it is well known [15], and in the turbulent fluid itself (with the index t) there are regions with an extremely strong "active" dissipation of turbulent energy $\varepsilon_d \gg \langle\varepsilon\rangle_t$, interspersed with regions where the dissipation is not so significant, $\varepsilon_{nd} \ll \langle\varepsilon\rangle_t$, $G_t = G_{td} + G_{tnd}$. This phenomenon is usual to call the "internal" intermittency, [15]. In this connection, in [2] were adopted new hypotheses of similarity.

Hypothesis 1. All the statements of the Kolmogorov statistical theory of the small-scale turbulence are valid only for the *dissipative* fluid of a turbulent flow.

Hypothesis 2. All the parameters and coefficients in the Kolmogorov theory are *statistical*.

According to the accepted hypotheses, the lognormal law of distribution of the values of the partially averaged dissipation of the turbulent energy ε_l is valid only for a dissipative fluid, while all the coefficients in this (dissipative) fluid are universal constants.

As it turns out, probability density function (*PDF*) of the random variable $\varepsilon_l = \varepsilon_l(\boldsymbol{x}, t)$ (as for any other random variable f) in the generalized region of a turbulent flow G is given by

$$P(\varepsilon_l) = \gamma_d P_d(\varepsilon_l) + \gamma_{nd} P_{nd}(\varepsilon_l) + (1-\gamma) P_n(\varepsilon_l) \quad (1)$$

where $\gamma_d, \gamma_{nd}, \gamma$ – intermittency factors as the probability of observing a dissipative, nondissipative and turbulent fluid in the region G, $\gamma_d = \gamma\gamma_{td}$, besides $\gamma_{nd} = \gamma(1-\gamma_{td})$, where γ_{td} – intermittency coefficient of a dissipative fluid inside a turbulent, i.e. in the region G_t.

According to *hypothesis 1*, conditional *PDF* value ε_l, i.e. *CPDF* $P_d(\varepsilon_l)$, for the dissipative fluid G_d takes the form

$$P_d(\varepsilon_l) = \frac{1}{\sqrt{2\pi}\sigma^2_{*ln\,\varepsilon_l}\,\varepsilon_l} exp\left(-\frac{\left(ln\,\varepsilon_l/\langle\varepsilon\rangle_d + \sigma^2_{*ln\,\varepsilon_l}/2\right)^2}{2\sigma^2_{*ln\,\varepsilon_l}}\right) \quad (2)$$

with the normalization condition $\int_0^\infty P_d(\varepsilon_l) d\,\varepsilon_l = 1$ and with dispersion $D_d\, ln\varepsilon_l = \sigma^2_{*ln\,\varepsilon_l}$, i.e. with variance of the logarithm of the random variable ε_l in the form of $\sigma^2_{*ln\,\varepsilon_l} = \langle(ln\,\varepsilon_l - \langle ln\,\varepsilon_l\rangle_d)^2\rangle_d$. Conditional statistical mean $\langle\varepsilon_l\rangle_d = \int_0^\infty \varepsilon_l P_d(\varepsilon_l) d\,\varepsilon_l$. It is considered in this case that the quantity $\langle\varepsilon_l\rangle_d$ depends on the size of the partial averaging l and that $\langle\varepsilon_l\rangle_d \cong \langle\varepsilon\rangle_d$ in case of $l = r_{min} \cong 10\eta$, where r_{min} – minimum scale of fluctuations (vortices) in the inertial interval, $\eta = 1/k_\eta$ – Kolmogorov scale, which, according to the data presented in [7], takes values from the range 0.04÷ 0.4mm depending on the value Re, $2625 \le Re_{м} \le 10^8$.

Modeling of conditional statistical moments of the partially averaged dissipation $\langle\varepsilon_l^n\rangle_d$ requires an appropriate expression for the variance $\sigma^2_{*ln\,\varepsilon_l}$ in (2). Such an expression, in view of *hypothesis 1*, coincides in form with the hypothetical Kolmogorov expression, i.e. for the dissipative fluid

$$\sigma^2_{*ln\,\varepsilon_l} = \langle\mu\rangle_d Ln(L_d/l) + A_d \quad (3)$$

where $\langle\mu\rangle_d = const$ and $A_d = const$. It follows that

$$\langle\varepsilon_l^n\rangle_d = C_{Ad}^{(n)} \langle\varepsilon_l\rangle_d^n (l/L_d)^{-\langle\mu\rangle_d n(n-1)/2} \quad (4)$$

where coefficients $C_{Ad}^{(n)} = e^{A_d n(n-1)/2} = const$ at the given value n. Now the expression for the total age$\langle \varepsilon_l^n \rangle = \gamma_d \langle \varepsilon_l^n \rangle_d$takes the form

$$\langle \varepsilon_l^n \rangle = C_{Ad}^{(n)} \gamma_d \langle \varepsilon \rangle_d^n (l/L_d)^{-\langle \mu \rangle_d n(n-1)/2} \quad (5)$$

by virtue of the fact that the dissipation in the non-dissipative G_{nd} and, especially, in the non-turbulent$G_n$flow regions is negligible, $\langle \varepsilon \rangle \cong \gamma \langle \varepsilon \rangle_t \cong \gamma_d \langle \varepsilon \rangle_d$because of$\langle \varepsilon \rangle_t \cong \gamma_{td} \langle \varepsilon \rangle_dand\gamma_d = \gamma \gamma_{td}$.

As a result, for longitudinal structure functions of the$n$-thorder in the dissipative fluid $G_d$we obtain an expression

$$S_d^{(n)}(r) = \langle v^n \rangle_d = C_{kd}^{(n)} \langle \varepsilon \rangle_d{}^{n/3} r^{n/3} \left(\frac{l}{L_d} \right)^{\zeta_d(n) - n/3} \quad (6)$$

whereL_d - integral scale of dissipative fluid flow, $C_{kd}^{(n)} = const$,

$$\zeta_d(n) = n/3 - \langle \mu \rangle_d n(n-3)/18 \quad (7)$$

The expression for the total statistical mean of the instantaneous hydrodynamic characteristics acquires, according to (1), the following form:

$$\langle f \rangle = \gamma_d \langle f \rangle_d + \gamma_{nd} \langle f \rangle_{nd} + (1-\gamma) \langle f \rangle_n \quad (8)$$

and is consistent with the expressions for the total and conditional (for turbulent fluid) mean[2],

$$\langle f \rangle = \gamma \langle f \rangle_t + (1-\gamma) \langle f \rangle_n, \ \ \langle f \rangle_t = \gamma_{td} \langle f \rangle_{td} + (1-\gamma_{td}) \langle f \rangle_{tnd} \quad (9)$$

According to *hypothesis 2*,coefficient μisa random variable with statistical total and conditional (in turbulent fluid) mean, so that from (9)we obtain:

$$\langle \mu \rangle = \gamma \langle \mu \rangle_t + (1-\gamma) \langle \mu \rangle_n, \ \ \langle \mu \rangle_t = \gamma_{td} \langle \mu \rangle_d + (1-\gamma_{td}) \langle \mu \rangle_{nd} \quad (10)$$

where coefficients$\langle \mu \rangle_d \equiv \langle \mu \rangle_{td}$ и $\langle \mu \rangle_{nd} \equiv \langle \mu \rangle_{tnd}$ are universal constants.

3. Testing of the lognormal distribution law of dissipation

The availability of experimental data [16, 17] for the values of "scaling" index $\zeta_d(n)$(7)(7), included in the expression of the structural functions $S_d^{(n)}(r)$(6)(6), allows us to estimate the validity of the lognormal law obtained by us, specifically to estimate *CPDFP*$_d(\varepsilon_l)$(2)(2)by testing the index $\zeta_d(n)$by selecting the appropriate value of coefficient $\mu = \langle \mu \rangle_d = const$.

Indeed, in the known models the structure of small-scale turbulence in the inertial interval of scales $\eta \ll r \ll L$is assumed to be locally isotropic (recall that the experimental data [4,5]do not confirm this). At that in the lognormal model *K-62*the quantity$\zeta(n) = n/3 - \mu n(n-3)/18$is included in the expression for the structure functions $S^{(n)}(r) \sim r^{\zeta(n)}$. It is precisely such a dependence that has been experimentally investigated in [16, 17]. The data obtained were used to test index $\zeta(n)$with the aim of estimating the validity of the Kolmogorov lognormal law by a suitable choice of the value of the coefficientμ.As a result, it was established, that the calculated curve $\zeta(n)$with value$\mu \geq 0.2$deviates from the experimental data at $n \gg 1$ (for$\mu = 0.3$the deviation begins at$n = 6$and increases with the growth of n).

Now we note that according to the *ASMTurbS* theory the structure of small-scale turbulence is locally isotropic only in the dissipative fluid of a turbulent flow. It follows that in our case $S_d^{(n)}(r){\sim}(l/L_d)^{\zeta_d(n)-n/3}$(6)(6), where $\zeta_d(n)$(7)(7) contains the coefficient$\mu = \langle\mu\rangle_d = const$.

InFig.1presented calculations$\zeta_n = \zeta_d(n)$, performed by formula(7)at different values of the cient$\mu = \langle\mu\rangle_d$(experimental data [16, 17] complies$Re_\lambda = 500 - 800$ and coincides with the data [13]in the region$2 \le n \le 10$). It can be seen that the calculated semi-empirical (due to the use of experimental data) curve 1 in case of $\langle\mu\rangle_d = 0.145$gives almost complete coincidence with the experimental data (results of calculations of$\beta$and *log-Poisson* modelssee, for example, in [8]).

The first result of our theory is the calculation of the exponential coefficient of crushing of vortices $\langle\mu\rangle_t$depending on the value of the coefficientγ.For this calculation requires, entering into (10),coefficient ues$\langle\mu\rangle_d$and$\langle\mu\rangle_{nd}$.In this case, the value of $\langle\mu\rangle_d = 0.145$has already been found. The value of the coefficient $\langle\mu\rangle_{nd}$can be estimated using the experimental data [3]. From these data and our expressions (10) follows that $\langle\mu\rangle_t = \langle\mu\rangle_{nd} \cong 0.32at\gamma \to 0$, because the probability of observing the turbulent fluid is low and hence the value of the coefficient $\gamma_{td} \to 0$.

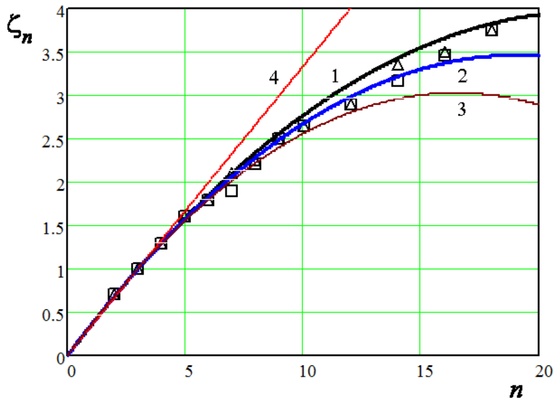
Calculation of coefficient $\langle\mu\rangle_t$according to the values of the coefficient γpresented in theFig.2. Also here presented the calculation of the total average$\langle\mu\rangle$with the condition $\langle\mu\rangle = \langle\mu\rangle_n = \langle\mu\rangle_{nd}at\gamma \to 0$. As it seen, the values $\langle\mu\rangle_t$are in good agreement with the experimental data [3], made for various types of turbulent flow. It is also seen that the coefficients $\langle\mu\rangle_d = 0.145and\langle\mu\rangle_{nd} \cong 0.32$can be considered universal constants.

Conclusion

So we have shown that the Kolmogorov theory is the foundation on which further effective development of the theory of small-scale turbulence is possible. In particular, allowance of the internal intermittency of dissipative fluid and the lognormal law proposed by us in the form of *CPDFP*$_d(\varepsilon_l)$(2)(2)allowed to determine both the universal value of the coefficient $\langle\mu\rangle_d = 0.145$ (just such a value was obtained experimentally in [3, 6]), and to find the dependence of the coefficient $\langle\mu\rangle_t$and $\langle\mu\rangle$on the value of the external intermittency coefficient γ. In doing so, we excluded some of the arbitrariness, designated in [7],in the choice of partial averaging size $l = r$, giving it a concrete value$l = r_{min} \cong 10\eta$. (The essence of this arbitrariness lies in the fact that the size of the partial averaging in the Kolmogorov *K-62* theory should be sufficiently small, i.e. $r{\sim}\eta$in order to$\langle\varepsilon_l\rangle \cong \langle\varepsilon\rangle$, whereas in the inertial range the value $r \gg \eta$, i.e. can assume high values, $r \gg 10\eta$).

In this connection, the results of calculations [2] require their refinement. For example, calculation of the Kolmogorov coefficient C_k в [2] for flows with a large number Re gives incredibly small value $r/L = 10^{-11}$ in case that$r/L_d = 10^{-7}$.

It follows that, to complete the *ASMTurbS*theory we still have to solve a number of problems associated with the determination of the basic statistical characteristics of small-scale turbulence. The main difficulty in this case is the determination of the relationship between the integral scalesL_d/L, since there are no data, including experimental data, for the integral scale of the dissipative fluid flow L_d. Moreover, the results presented by us indicate the need for a detailed experimental study of the small-scale structure precisely for the dissipative fluid of a turbulent flow.



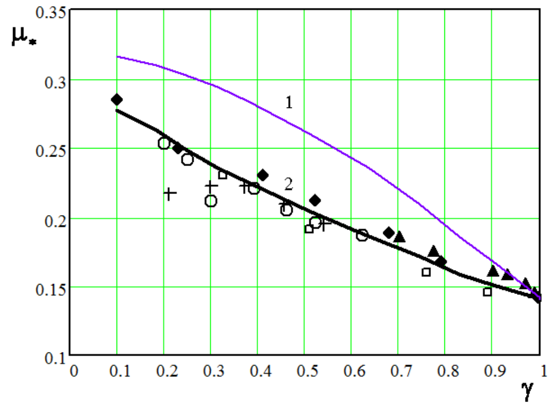


Fig.1. Scaling indicators ζ_n for the structure functions of longitudinal velocity.Solid lines - calculation at $\langle\mu\rangle_d = 0.145$ (1), $\langle\mu\rangle_d = 0.17$ (2), $\langle\mu\rangle_d = 0.2$ (3), $\langle\mu\rangle_d = 0$ (4) – corresponds to the theory *K-41*;icons - experimental data Δ – [16], □ – [17].

Fig.2. Distribution of values of coefficients μ_*depending on the value of the coefficient of external intermittency γ. Solid lines - calculation $\mu_* = \langle\mu\rangle$ (1), $\mu_* = \langle\mu\rangle_t$(2);icons - experimental data [3].

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