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A Note on Sum Square Difference Product Prime Labeling

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3

***Abstract*— Sum square difference product prime labeling of a graph is the labeling of the vertices with {0,1,2---------,p-1} and the edges with absolute difference of the square of the sum of the labels of the incident vertices and product of the labels of the incident vertices. The greatest common incidence number of a vertex (*gcin*) of degree greater than one is defined as the greatest common divisor of the labels of the incident edges. If the *gcin* of each vertex of degree greater than one is one, then the graph admits sum square difference product prime labeling. Here we identify friendship graph, double graph of a star, splitting graph of a star, splitting graph of a path, tadpole graph, triangular book, key graph etc for sum square difference product prime labeling.**

Keywords*— Graph labeling; greatest common incidence number; sum square; product; prime labeling.

I. Introduction

All graphs in this paper are finite, undirected and simple. The symbol V(G) and E(G) denotes the vertex set and edge set of a graph G. The graph whose cardinality of the vertex set is called the order of G, denoted by p and the cardinality of the edge set is called the size of the graph G, denoted by q. A graph with p vertices and q edges is called a (p,q)- graph.

A graph labeling is an assignment of integers to the vertices or edges. Some basic notations and definitions are taken from [2],[3] and [4] . Some basic concepts are taken from [1] and [2]. In [5], we introduced the sum square difference product prime labeling and proved the result for some path related graphs. In this paper we investigated sum square difference product prime labeling of friendship graph, double graph of a star, splitting graph of a star, splitting graph of a path, tadpole graph, triangular book, key graph etc.

Definition: 1.1 Let G be a graph with p vertices and q edges. The greatest common incidence number (***gcin***) of a vertex of degree greater than or equal to 2, is the greatest common divisor (gcd) of the labels of the incident edges.

II. MAIN RESULTS

Definition 2.1 Let G = (V(G),E(G)) be a graph with p vertices and q edges .Define a bijection f : V(G) → {0,1,2,3,----------------,p-1} by $f(v_i) = i-1$, for every i from 1 to p and define a 1-1 mapping f_{ssdppl}^{*} : E(G) → set of natural numbers N by

$f_{ssdppl}^{*}(uv)$ = $|\{f(u) + f(v)\}^2 - f(u)f(v)|$.

The induced function f_{ssdppl}^{*} is said to be sum square difference product prime labeling, if for each vertex of degree at least 2, the greatest common incidence number is 1.

Definition 2.2 A graph which admits sum square difference product prime labeling is called a sum square difference product prime graph.

Theorem 2.1 Friendship graph F_n, admits sum square difference product prime labeling.

Proof: Let G = F_n and let v_1, v_2,----------------,v_{2n+1} are the vertices of G

Here |V(G)| = 2n+1 and |E(G)| = 3n

Define a function f : V → {0,1,2,3,----------------,2n } by

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$f(v_i) = i-1$, i = 1,2,------,2n+1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(v_1\, v_{2i})$ = $(2i-1)^2$, i = 1,2,------------,n

$f^*_{ssdppl}(v_1\, v_{2i+1})$ = $(2i)^2$, i = 1,2,------------,n

$f^*_{ssdppl}(v_{2i}\, v_{2i+1})$ = $12i^2-6i+1$, i = 1,2,------------,n

Clearly f^*_{ssdppl} is an injection.

gcin of (v_1) = 1

gcin of (v_{2i}) = gcd of $\{f^*_{ssdppl}(v_1\, v_{2i}), f^*_{ssdppl}(v_{2i}\, v_{2i+1})\}$

= gcd of $\{(2i-1)^2, 12i^2-6i+1\}$

= gcd of $\{2i-1, 12i^2-6i+1\}$

= gcd of $\{(2i-1), (2i-1)6i+1\}$

= 1 , i = 1,2,------------,n

gcin of (v_{2i+1}) = gcd of $\{f^*_{ssdppl}(v_1\, v_{2i+1}), f^*_{ssdppl}(v_{2i}\, v_{2i+1})\}$

= gcd of $\{(2i)^2, 12i^2-6i+1\}$

= gcd of $\{2i, 12i^2-6i+1\}$

= gcd of $\{(2i), (6i-3)2i+1\}$

= 1 , i = 1,2,------------,n

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence F_n, admits sum square difference product prime labeling. ■

Example 2.1 G = F_3

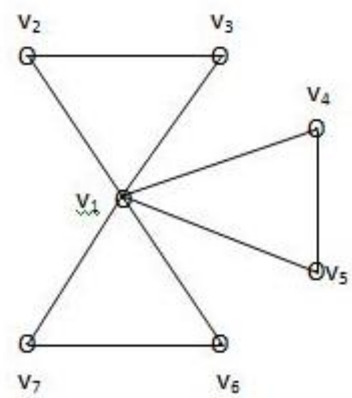

fig- 2.1

Theorem 2.2 Double graph of star $K_{1,n}$, admits sum square difference product prime labeling.

Proof: Let G = $D(K_{1,n})$ and let a, b, v_1,v_2,--------------,v_n,u_1,u_2,--------------,u_n are the vertices of G.

Here |V(G)| = 2n+2 and |E(G)| = 4n

Define a function f : V → {0,1,2,3,----------------,2n+1 } by

$f(v_i) = i+1$, i = 1,2,------,n

$f(u_i) = n+ i+1$, i = 1,2,------,n

f(a) = 0, f(b) = 1.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(av_i)$ = $(i+1)^2$, i = 1,2,------------,n

$f^*_{ssdppl}(bv_i)$ = i^2+3i+3, i = 1,2,------------,n

$f^*_{ssdppl}(au_i)$ = $(n+i+1)^2$, i = 1,2,------------,n

$f^*_{ssdppl}(bu_i)$ = $(n+i+1)^2+(n+i+1)+1$ i = 1,2,------------,n

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Clearly f^*_{ssdppl} is an injection.

gcin of (a) = 1.
gcin of (b) = 1.
gcin of (v_i) = gcd of $\{f^*_{ssdppl}(av_i), f^*_{ssdppl}(bv_i)\}$
= gcd of $\{(i+1)^2, i^2+3i+3\}$
= gcd of $\{(i+1), (i+1)(i+2)+1\}$= 1, $i = 1,2,$------------,n
gcin of (u_i) = gcd of $\{f^*_{ssdppl}(au_i), f^*_{ssdppl}(bu_i)\}$
= gcd of $\{(n+i+1)^2, (n+i+1)^2 + (n+i+1)+1\}$
= gcd of $\{(n+i+1), (n+i+1)^2 + (n+i+1)+1\}$
= 1, $i = 1,2,$------------,n

So, ***gcin*** of each vertex of degree greater than one is 1.
Hence $D(K_{1,n})$, admits sum square difference product prime labeling. ■

Example 2.2 G = $D(K_{1,4})$

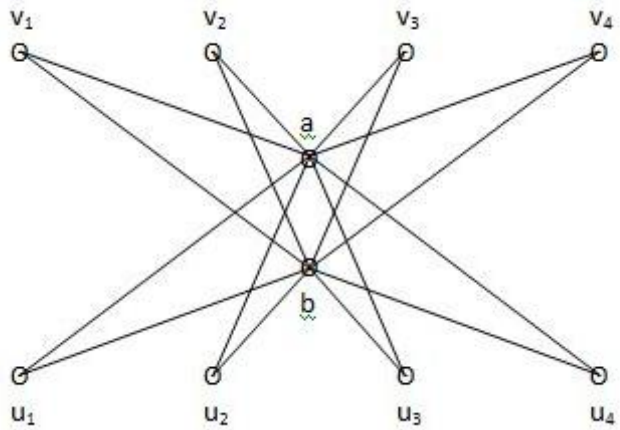

fig- 2.2

Theorem 2.3 Splitting graph of star $K_{1,n}$, admits sum square difference product prime labeling.

Proof: Let G = $S'(K_{1,n})$ and let a, b, $v_1,v_2,$--------------,$v_n,u_1,u_2,$--------------,u_n are the vertices of G

Here |V(G)| = 2n+2 and |E(G)| = 3n

Define a function f : V → {0,1,2,3,----------------,2n+1 } by
$f(v_i) = i+1$, $i = 1,2,$------,n
$f(u_i) = n+i+1$, $i = 1,2,$------,n
f(a) = 0, f(b) = 1.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows
$f^*_{ssdppl}(av_i)$ = $(i+1)^2$, $i = 1,2,$------------,n
$f^*_{ssdppl}(bv_i)$ = i^2+3i+3, $i = 1,2,$------------,n
$f^*_{ssdppl}(au_i)$ = $(n+i+1)^2$, $i = 1,2,$------------,n

Clearly f^*_{ssdppl} is an injection.

gcin of (a) = 1.
gcin of (b) = 1.
gcin of (v_i) = gcd of $\{f^*_{ssdppl}(av_i), f^*_{ssdppl}(bv_i)\}$
= gcd of $\{(i+1)^2, i^2+3i+3\}$
= gcd of $\{(i+1), (i+1)(i+2)+1\}$= 1, $i = 1,2,$------------,n

So, ***gcin*** of each vertex of degree greater than one is 1.

ISSN: 2349-3224 || www.ijaetmas.com || Volume 04 - Issue 09 || September-2017 || PP. 50-56

Hence S'($K_{1,n}$) , admits sum square difference product prime labeling. ■

Example 2.3 G = S'($K_{1,4}$)

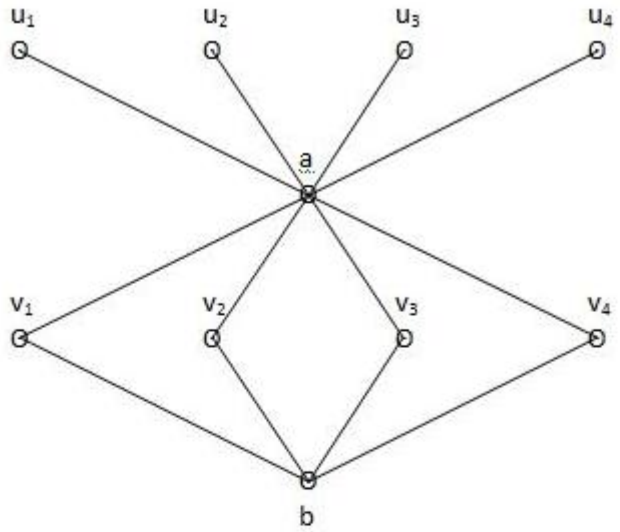

fig – 2.3

Theorem 2.4 Splitting graph of path P_n admits sum square difference product prime labeling, when n is odd and (n+4) $\not\equiv$ 0(mod7) and n $\not\equiv$ 0(mod7).

Proof: Let G = S'(P_n) and let v_1,v_2,----------------,v_{2n} are the vertices of G

Here |V(G)| = 2n and |E(G)| = 3n-3

Define a function f : V → {0,1,2,3,----------------,2n-1 } by

$$f(v_i) = i-1 , i = 1,2,------,2n$$

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(v_i\, v_{i+1})$ = $3i^2$-3i+1, i = 1,2,------------,n-1

$f^*_{ssdppl}(v_{n+i}\, v_{n+i+1})$ = $(2n+2i-1)^2$-(n+i)(n+i-1), i = 1,2,------------,n-1

$f^*_{ssdppl}(v_{2i-1}\, v_{2i+5})$ = $12i^2$+12i+12, i = 1,2,------------,$\frac{n-1}{2}$

$f^*_{ssdppl}(v_{2i+1}\, v_{2i+5})$ = $12i^2$+24i+16, i = 1,2,------------,$\frac{n-1}{2}$

Clearly f^*_{ssdppl} is an injection.

gcin of (v_1) = gcd of {$f^*_{ssdppl}(v_1\, v_2)$, $f^*_{ssdppl}(v_1\, v_{n+2})$ }
= gcd of {1,$(n+1)^2$} = 1

gcin of (v_{i+1}) = gcd of {$f^*_{ssdppl}(v_i\, v_{i+1})$, $f^*_{ssdppl}(v_{i+1}\, v_{i+2})$ }
= gcd of { $3i^2$-3i+1, $3i^2$+3i+1}
= gcd of {6i , $3i^2$-3i+1}
= gcd of {3i , $3i^2$-3i+1}
= gcd of {3i , 3i(i-1)+1}
= 1, i = 1,2,------------,n-2

gcin of (v_{n+i+1}) = 1, i = 1,2,------------,n-2

gcin of (v_n) = gcd of {$f^*_{ssdppl}(v_{n-1}\, v_n)$, $f^*_{ssdppl}(v_{2n-1}\, v_n)$ }
= gcd of { $3n^2$-9n+7, $7n^2$-14n+7}
= 1

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence S'(P_n), admits sum square difference product prime labeling. ■

ISSN: 2349-3224 || www.ijaetmas.com || Volume 04 - Issue 09 || September-2017 || PP. 50-56

Example 2.4 G = S'(P_5)

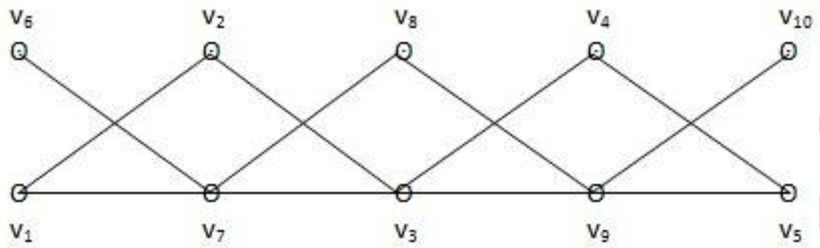

fig – 2.4

Theorem 2.5 Triangular book $P_2 + \overline{K_n}$, admits sum square difference product prime labeling.

Proof: Let G = $P_2 + \overline{K_n}$ and let a, b, v_1,v_2,--------------,v_n are the vertices of G

Here |V(G)| = n+2 and |E(G)| = 2n+1

Define a function f : V → {0,1,2,3,----------------,n+1 } by

$f(v_i) = i+1$, i = 1,2,------,n

f(a) = 0, f(b) = 1.

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(av_i) = (i+1)^2$, i = 1,2,------------,n

$f^*_{ssdppl}(bv_i) = i^2+3i+3$, i = 1,2,------------,n

$f^*_{ssdppl}(ab) = 1$

Clearly f^*_{ssdppl} is an injection.

gcin of (a) = 1.

gcin of (b) = 1.

gcin of (v_i) = gcd of {$f^*_{ssdppl}(av_i)$, $f^*_{ssdppl}(bv_i)$ }

= 1, i = 1,2,------------,n

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $P_2 + \overline{K_n}$,admits sum square difference product prime labeling. ■

Example 2.5 G = $P_2 + \overline{K_4}$

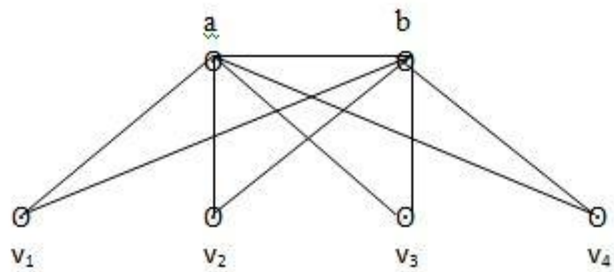

fig – 2.5

ISSN: 2349-3224 || www.ijaetmas.com || Volume 04 - Issue 09 || September-2017 || PP. 50-56

Theorem 2.6 Tadpole graph $C_n(P_m)$ admits sum square difference product prime labeling when n is odd.

Proof : Let G = $C_n(P_m)$ and let v_1, v_2, --------------,v_{m+n-1} are the vertices of G.

Here |V(G)| = n+m-1 and |E(G)| = n+m-1

Define a function f : V → {0,1,2,3,----------------, n+m-2 } by

$f(v_i) = i-1$, i = 1,2,------, n+m-1

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(v_i\, v_{i+1}) = 3i^2-3i+1$, i = 1,2,------------,n+m-2

$f^*_{ssdppl}(v_1\, v_n) = (n-1)^2$.

gcin of (v_1) = gcd of $\{f^*_{ssdppl}(v_1\, v_2), f^*_{ssdppl}(v_1\, v_n)\}$

= gcd of $\{1,(n-1)^2\}$ = 1.

gcin of (v_{i+1}) = 1, i = 1,2,------------,n+m-3

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence $C_n(P_m)$, admits sum square difference product prime labeling. ■

Example 2.6 G = $C_5(P_4)$

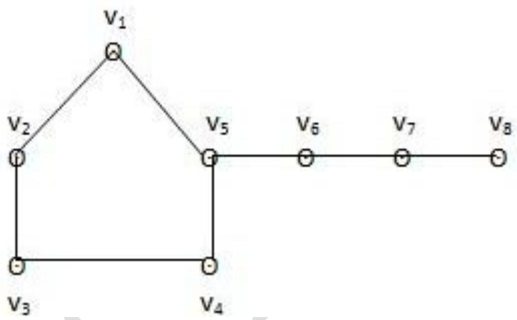

fig – 2.6

Theorem 2.7 Key graph K(n,m) admits sum square difference product prime labeling when n is odd.

Proof : Let G = K(n.m) and let v_1, v_2, --------------,v_{2m+n-2} are the vertices of G.

Here |V(G)| = n+2m-2 and |E(G)| = n+2m-2

Define a function f : V → {0,1,2,3,----------------, n+2m-3 } by

$f(v_i) = i-1$, i = 1,2,------, n+2m-2

Clearly f is a bijection.

For the vertex labeling f, the induced edge labeling f^*_{ssdppl} is defined as follows

$f^*_{ssdppl}(v_i\, v_{i+1}) = 3i^2-3i+1$, i = 1,2,------------,n+m-1

$f^*_{ssdppl}(v_1\, v_n) = (n-1)^2$.

$f^*_{ssdppl}(v_{n+i}\, v_{n+2m-i-1}) = (2n+2m-3)^2-(n+2m-i-2)(n+i-1)$, i = 1,2,------------,m-2

gcin of (v_1) = gcd of $\{f^*_{ssdppl}(v_1\, v_2), f^*_{ssdppl}(v_1\, v_n)\}$

= gcd of $\{1,(n-1)^2\}$ = 1.

gcin of (v_{i+1}) = 1, i = 1,2,------------,n+m-2

So, ***gcin*** of each vertex of degree greater than one is 1.

Hence K(n,m) , admits sum square difference product prime labeling. ■

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ISSN: 2349-3224 || www.ijaetmas.com || Volume 04 - Issue 09 || September-2017 || PP. 50-56

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