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Topographic optimization using dynamic stiffness for a plate

1st Geisiel Moreira de Assis, 2nd Bruno Cesar Pockszevnicki[2],
3rd Thiago Fonseca Meirelles[3], 4th Helder Fontes
[1]Mechanical Engineering &PUC MINAS, Brazil
[2]Mechanical Engineering & PUC MINAS, Brazil
[3]Mechanical Engineering & CEFET, Brazil
[4]Mechanical Engineering & Pitágoras, Brazil

***Abstract*—In this work, a structural optimization strategy was studied in order, to change the shape of metal component plates used for example in the vehicle body. To this end, we first sought to validate the methodology employed in a plate by verifying its dynamic response in terms of the first vibrational modes as well as its response in the frequency domain, thus evaluating its dynamic stiffness. The study therefore comprises static and dynamic stiffness evaluation as well as evaluation of the natural frequency response for the free situation without damping and a natural frequency response for a forced excitation without damping. The objective of the study is to conduct geometric modifications by optimization software in order to improve the structural behavior for sheet metals with the aim improve the structural behavior of vehicular bodies through topographic optimization.**

Keywords*— Topographic Optimization; Dynamic Rigidity; Inertance; Modal Analysis; FRF;

I. Introduction

The Finite Element Method (FEM) is an important computational tool that is used in the solution of contour value problems described by partial differential equations by subdividing the geometry of the problem into smaller elements, and the approximation of the exact solution, can be obtained by interpolation of an approximate solution [1].

Currently, the method is applied in practically all engineering areas, such as: stress – strain analysis, heat transfer, fluid mechanics, among others. This methodology together with optimization algorithms allows optimized solutions for dimensional, shape or topology for structures in product development.

This work consists in the evaluation of a plate in balance with the application of the MEF to obtain: the natural frequency modes, the rigidity of this structure and a study of the inertance for a frequency range between 1 and 1000Hz. The aim is to provide a way to optimize sheet metals in body structure of vehicles. Size of mesh used in simulations is the same normally that is used for vehicular bodies in structural evaluation about 3mm, and it was used shell for element type.

The study aims to give an overview of the methodology of topographic optimization, which consists in increasing structural rigidity and comparing the results to the evaluated proposals.

To evaluate the modelling and applications of finite element method and topographic optimization, modal analyses were conducted and compared results with original concept.

This methodology could be implement and other development of vehicle components.

II. Model Description

A. *Optimization*

In the optimization philosophy, a complex decision problem is addressed, which is to find a better solution of all possible solutions. It can be divided into two categories depending on

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whether the variables are continuous or discrete. Solution methods for discrete optimization problems are generally classified into a combinatorial and continuous approaches [2].

The aim of this approaches can be an integer, a permutation or graph from a finite (or possibly countably infinite) set of solutions. And to solve an optimization problem strategies to formulate are necessary as linear and non-linear programming.

The objective is always maximized (or minimized, depending on the formulation) subject to the constraints that may limit the selection of decision variable values[3].

In terms of structural optimization, it could be used topology (The topology optimization method solves the basic engineering problem of distributing a limited amount of material in a design space [4]), topography (Topography optimization is an advanced form of shape optimization in which a design region for a given part is defined and a pattern of shape variable-based reinforcements within that region is generated using OptiStruct [5])or size (Size optimization defines ideal component parameters, such as material values, cross-section dimensions and thicknesses).

In linear programming, the optimization problem is characterized, as the name implies, by linear functions of the unknowns; the objective is linear in the unknowns, and the constraints are linear equalities or linear inequalities in the unknowns [6]. For example:

Minimize or Maximize:

$$x = f(x)$$

Subject to:

$$g_i(x) \leq 0, \qquad i = 1, \ldots, m$$
$$h_i(x) = 0, \qquad i = 1, \ldots, p$$

Where: $f(x)$ is objective function to be minimized over the variable x, $g_i(x)$ are inequality constraints and $h_i(x)$are equality constraints. In the case of this study will be minimize nodal displacement at grid point where loading is applied, subject to shape variables generated automatically on the designable space aligned with the elements normal, and introducing manufacturing constraints in the optimization process, such as minimum width and minimum draw height.

B. Frequency Response Function Overview

For performing vibration analysis and testing, one of many tools available is the frequency response function (FRF), which is a transfer function, expressed in the frequency domain. A FRF expresses the structural response to an applied force as a function of frequency. The response may be given in terms of displacement, velocity, or acceleration [7],[8]. The model of transfer function is presented in figure 1.

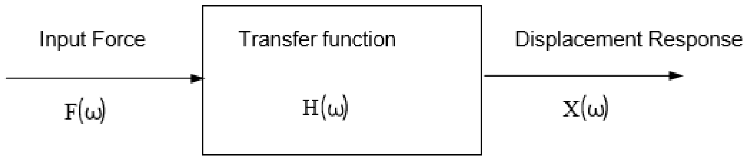

Fig. 1 Diagram of linear system used in frequency response function

Where:

F(ω) is the input force as a function of the angular frequency ω;

H(ω) is the transfer function.

X(ω) is the displacement response function.

Each function is a complex function, which may also be represented in terms of magnitude and phase. Each function is thus a spectral function that could be solved by Fourier transform.

$$X(\omega) = F(\omega).H(\omega)$$
$$H(\omega) = {}^{X(\omega)}\!/_{F(\omega)} \qquad \text{Eq. 1}$$

And similar transfers functions can be developed for the velocity and accelerationsfor the velocity and acceleration responses.For example: a single-degree-of-freedom system with damping and excitation force given by dynamic equation below [9][10]:

$$m\ddot{x} + c\dot{x} + Kx = F$$
$$\ddot{x} + \frac{c}{m}\dot{x} + \frac{K}{m}x = \frac{F}{m} \qquad \text{Eq. 2}$$

m, mass; c viscous damping coefficient, K stiffness, x absolute displacement of mass and F applied force, being:

$${}^{c}\!/_{m} = 2\xi\omega_n \qquad \text{Eq. 3}$$
$${}^{K}\!/_{m} = {\omega_n}^2 \qquad \text{Eq. 4}$$

where $\omega_n$is the natural frequency in (radians/sec), and ξ is the damping ratio, it is possible to substituting the terms into dynamic equation:

$$\ddot{x} + 2\xi\omega_n\dot{x} + {\omega_n}^2x = {\omega_n}^2\frac{F}{K} \qquad \text{Eq. 5}$$

The Fourier transform of each side of equation (5) may be taken to derive the steady-state transfer function for the absolute response displacement, as shown:

$$\frac{X(\omega)}{F(\omega)} = \frac{{\omega_n}^2}{K(\omega_n^2 - \omega^2 + i(2\xi\omega\omega_n)^2} \qquad \text{Eq. 6}$$

This transfer function, which represents displacement over force, is sometimes called thereceptance function, and in the same way, when a transfer function is used in terms of acceleration, we have:

$$\frac{\ddot{X}(\omega)}{F(\omega)} = \frac{-\omega^2\omega_n^2}{K(\omega_n^2 - \omega^2 + i(2\xi\omega\omega_n)^2} \qquad \text{Eq. 7}$$

which is called acceleration function or inertance. Another important relationship is dynamic stiffness given by:

$$\frac{F(\omega)}{X(\omega)} = m\sqrt{(\omega_n^2 - \omega^2)^2 + (2\xi\omega\omega_n)^2} \qquad \text{Eq. 8}$$

In figure 1 the model of the set plate with the identification of the restrictions in the model

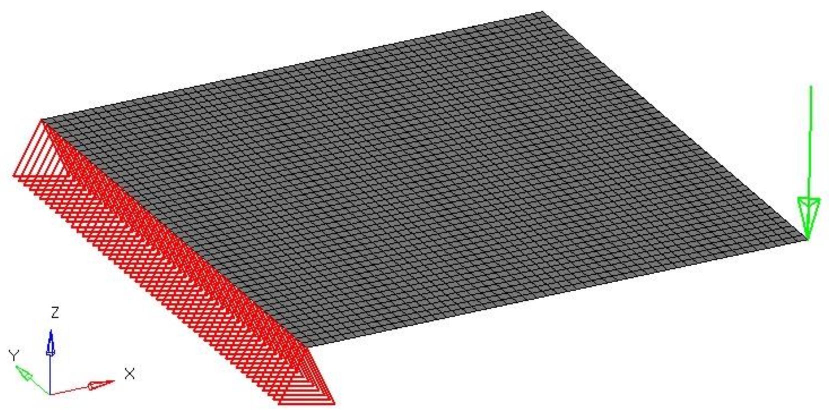

Fig. 2 Finite element plate used for simulations

C. Modeling Plate Problem

The six degrees of freedom were considered restricted to one end of the plate for all analysis and in the evaluation of stiffness, a unit load [1N] was applied. The Finite Element

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model of the plate was constructed using elements of the 2D shape (shell) to represent the plate in balance. It was used for the shell elements, the quadrilateral (CQUAD4) under the conditions of 1st order.

The plate has dimensions of 100x100x1,0mm and the material used is a linear steel with the properties described in Table I.

TABLE I
MECHANICAL PROPERTIES OF STEEL

Elasticity Modulus [MPa]	Especif mass [ton/mm³]	Poisson
210000	7,8E-9	0,30

The softwares used for the analyzes were: Altair HyperWorks V10.0®.

D. Inertance Procedure

The plate inertance study was evaluated for a frequency range between 1 and 1000Hz to identify the critical points of the structure. With the use of the OptiStruct solver some parameters were defined for extraction of this scanning interval.

As input conditions in the software, it was defined as the first frequency in the set being 1Hz, and an increment of 1Hz was adopted for the evaluated interval. As a response to the calculation we have the amplitude of the nodal displacement and the dynamic acceleration, both of which are in function of the frequency, to obtain the stiffness curve of the structure.

III. RESULTS OF THE INITIAL PROPOSAL

In view of the conditions mentioned above for the analysis of natural frequency, stiffness and inertia, we have below the results for the crimped plate.

E. Natural Frequency Analysis

Figures 3 and 4 show the first two vibration modes of the crimped plate.

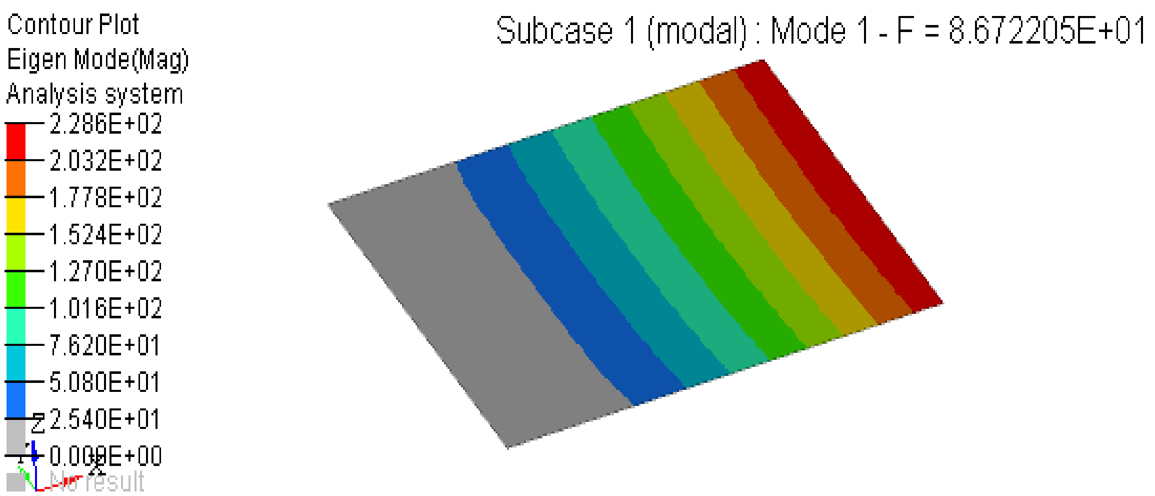

Fig. 3 First and second vibrations mode of plate

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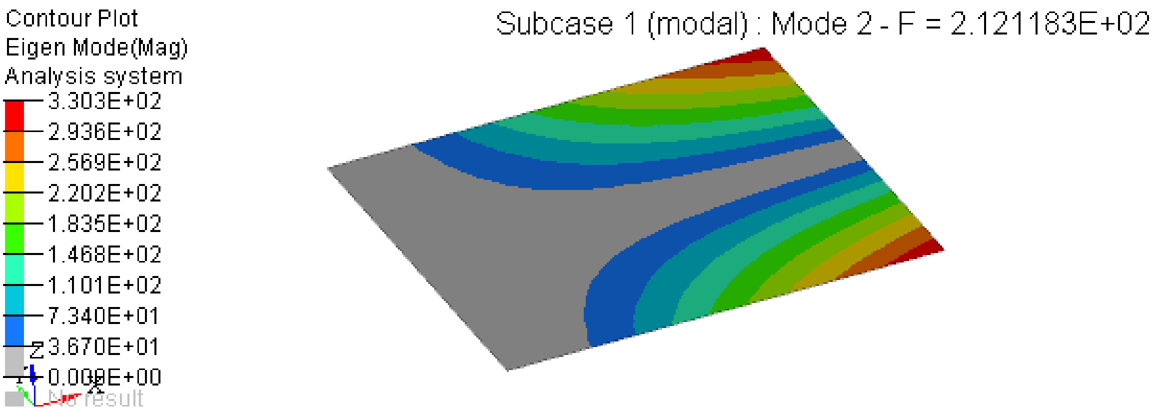

Fig. 4 First and second vibrations mode of plate

F. Rigidity analysis

In Fig. 5 we have the results of the rigidity of the plate.The stiffness value was obtained by dividing the force applied by the displacement.

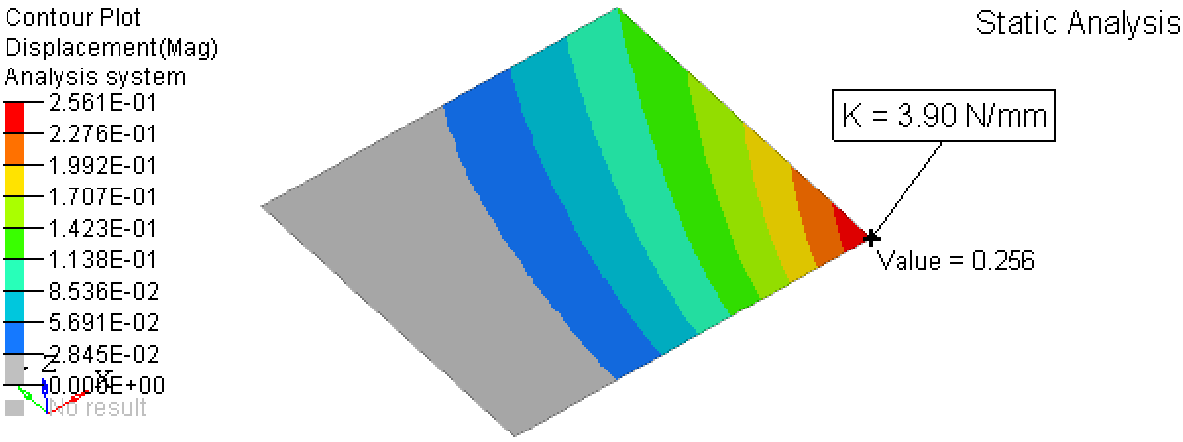

Fig. 5 Stiffness of plate

G. Inertance evaluation

Fig.6 shows the overall displacement of the structure as a function of the evaluated frequency range.

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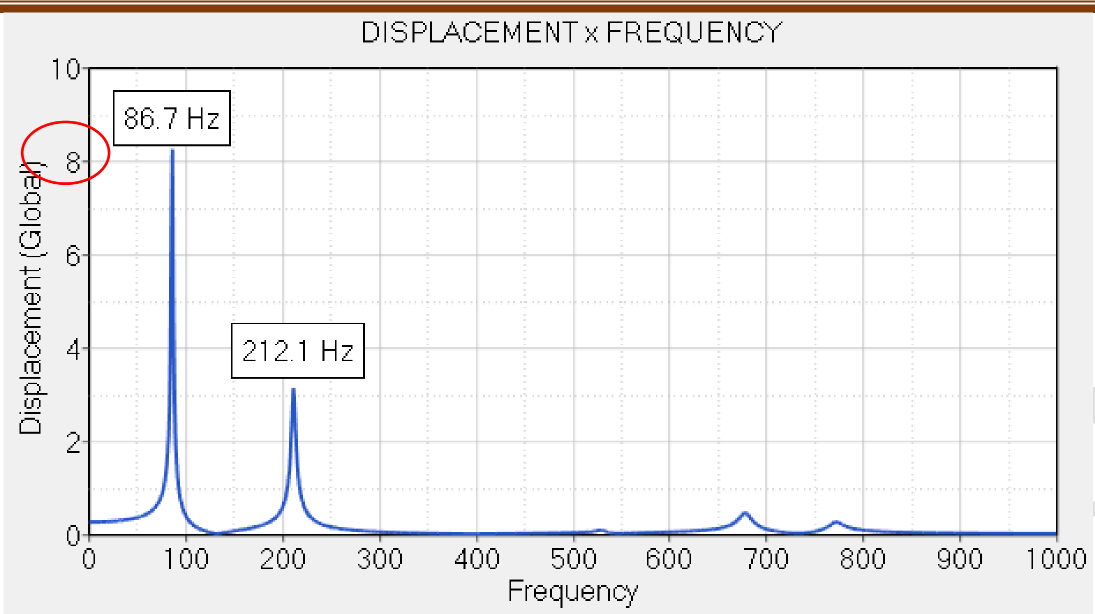

Fig. 6 Inertance Response

In figure 7 we have the response of the dynamic acceleration of the structure as a function of the frequency range evaluated and a stiffness curve (10N / mm) as the reference of analysis.

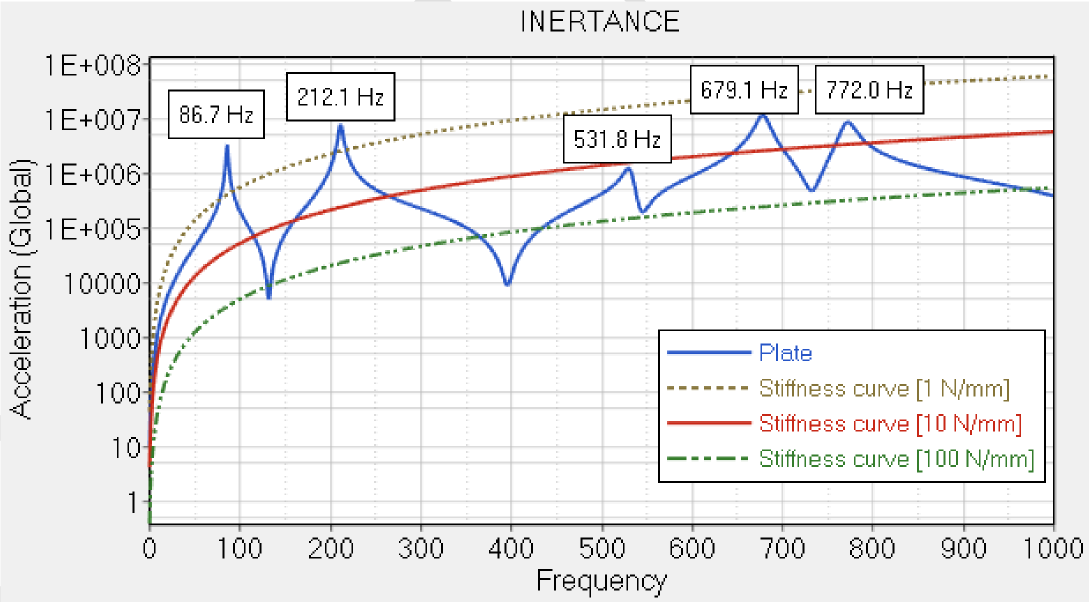

Fig. 7 Inertance Response with isocurve of dynamic stiffness

H. Topographic Optimization

This technique applies specifically to the design of plate and shell reinforcers. It consists of finding the distribution of a reinforcing pattern in the structures of plates and shells.

Defining project variables

This methodology uses the concept of parametric optimization in the sense that it does not modify the geometry of the reinforcer, only its aspect ratio along the plate, in the sense that it finds the optimum topology of the reinforcer along the structure. Fig. 8 shows a schematic of the control variables of this analysis.

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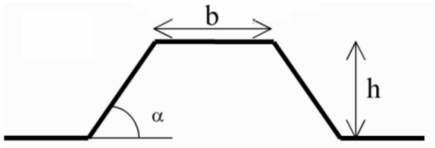

Fig. 8- Reinforcer used in topographic optimization

The solutions are based on the dimensional definition of the width, angle and height for the elements. The type of response and objectives for the optimization are also defined. The response of optimization is strongly dependent of these parameters, which normally are influenced by the fabrication process.

In this study the definitions of the variables are presented in Table II and adopted as objective of response the static displacement of the structure.

TABLE II
DESIGN VARIABLES FOR TOPOGRAPHIC OPTIMIZATION

Minimum Width [mm]	**Maximum Angle [mm]**	**Maximum Height** [mm]
5,0	60,0	4,0

In figure 9 we have the results of topographic optimization.

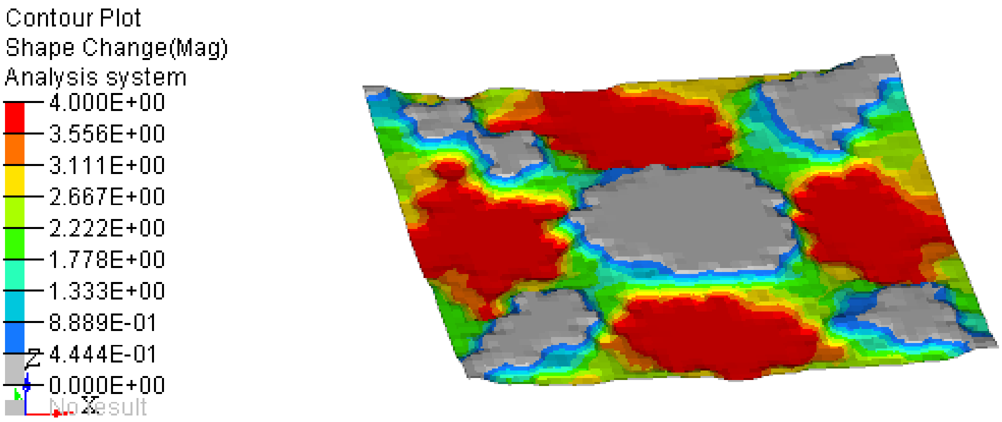

Fig. 9- Result of topographic optimization

1) *Results of The Optimized Proposal:*

Based on the optimization response, the natural frequency, stiffness and inertia modes were reassessed considering the same contour conditions. The refinement of mesh was considered around 3mm. If the refinement is reduced to 1.5mm the line of orientation of the rib becomes more pronounced, demonstrating a dependence with respect to the parameter width.

1.1) Natural Frequency Analysis: Figures 10 and 11 shows the first two vibration modes of the optimized plate.

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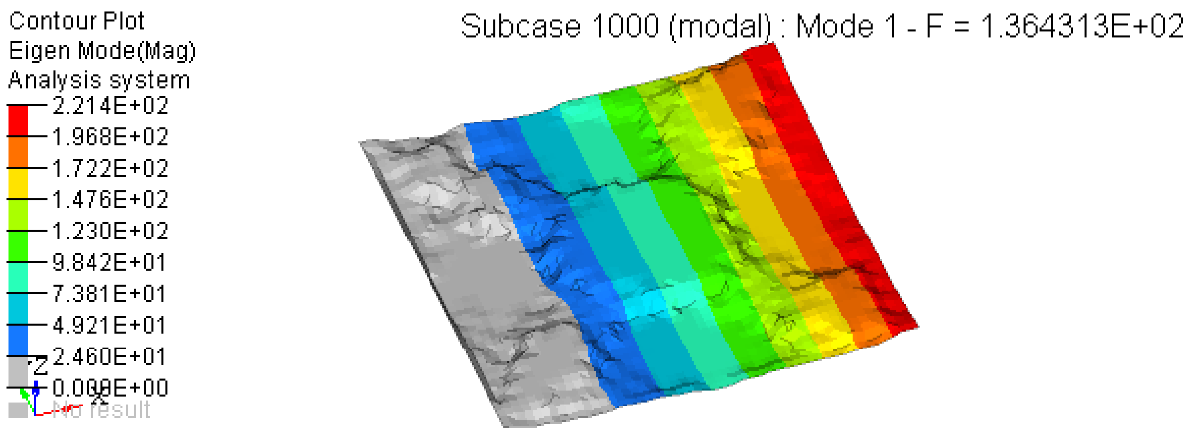

Fig. 10 First vibration mode of optimized plate.

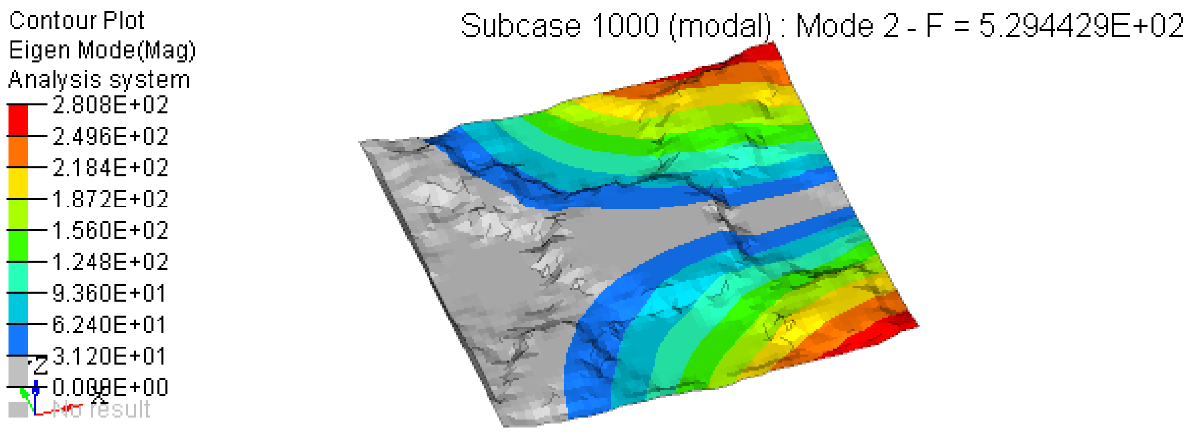

Fig. 11Second vibration mode of optimized plate.

2) Results of The Optimized Proposal

Based on the optimization response, the natural frequency, stiffness analysis was performed.In figure 12 we have the results of the optimized plate stiffness.

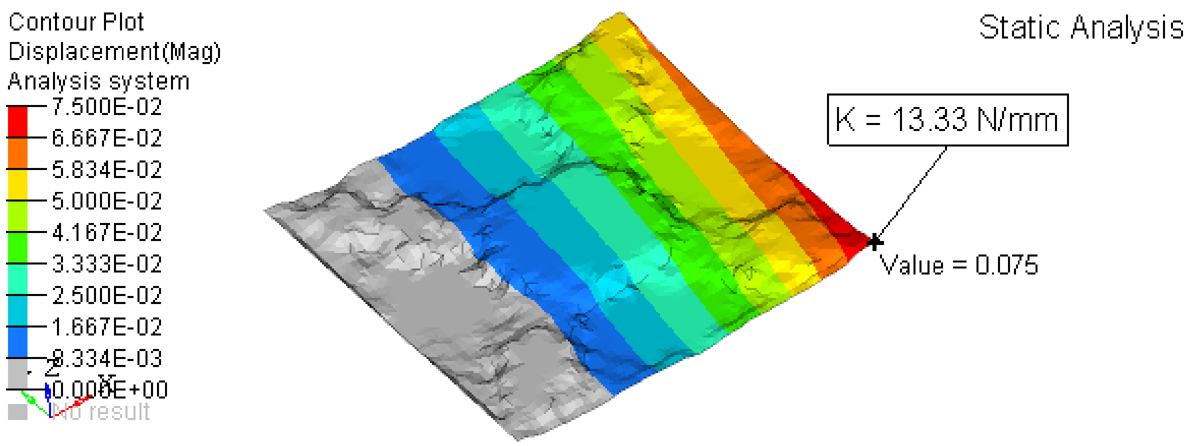

Fig. 12 Stiffness of optimized plate

3) *Inertance analysis*

Figure 13 shows the overall displacement of the optimized structure as a function of the evaluated frequency range.

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In order, to calculate the inertance, the FRF is obtained from the solution of finite element method applied for modal analysis, setting frequency response analysis to calculate the response of a structure under a harmonic excitation, ranging since 1Hz to 1000Hz. The inertance is evaluated and its accelerations are plotted in the frequency domain. To compare the effect of inertance an allowable curve is stablished called isocurve of dynamic stiffness curve which enable evaluate the stiffness in frequency domain.

The dynamic stiffness is obtained from the representation of the equilibrium position X of the vibrational system according to equation 9:

$$X(t) = A.sen(\omega t + \varphi) \qquad \text{Eq. 9}$$

The velocity and the acceleration are given by:

$$\dot{X}(t) = A\omega.cos(\omega t + \varphi) \qquad \text{Eq. 10}$$

$$\ddot{X}(t) = -A\omega^2.sen(\omega t + \varphi) \qquad \text{Eq. 11}$$

and if we evaluate the amplitude of displacement obtained in a system given by:

$$X(t) = {}^{F}/_{K} \qquad \text{Eq. 12}$$

where:

A – amplitude of displacement; $X(t)$,$\dot{X}(t)$,$\ddot{X}(t)$, displacement, velocity and acceleration function of time respectively; ω, angular velocity (ω =2πf); φ, phase angle, and f frequency of vibrational system. It is possible to describe acceleration in terms of:

$$\ddot{X}(t) = F\frac{(2\pi f)^2}{K} \qquad \text{Eq. 13}$$

Figure 13 shows the response of Frequency Response Function (FRF).

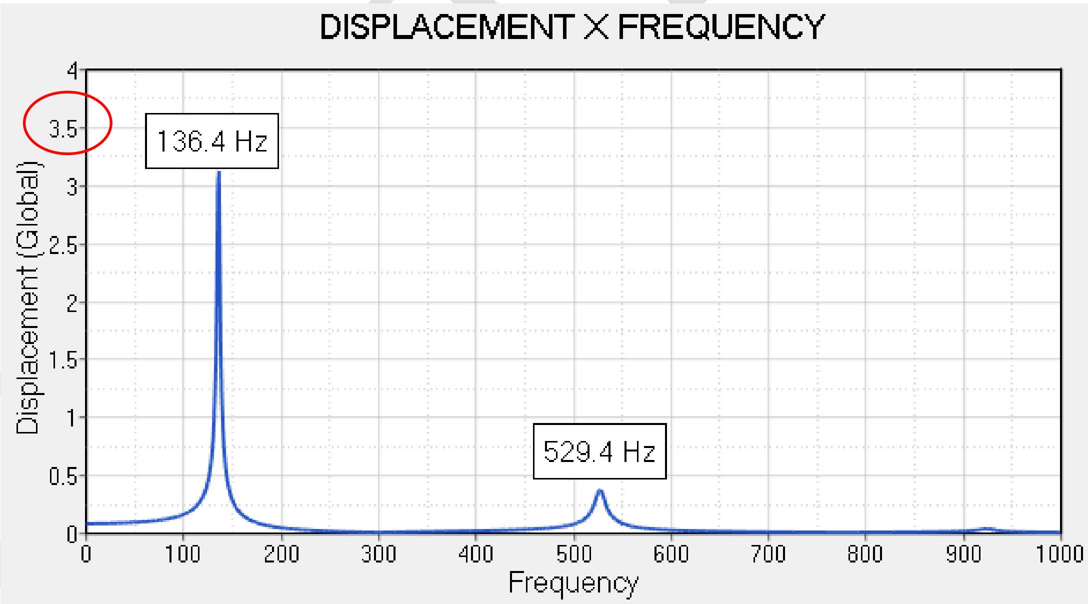

Fig. 13 Stiffness of optimized plate

I. Comparison of Results

In table 3 are presented a comparison between the results of the natural frequency and stiffness analyses for the models of the initial condition and optimized proposal.

From this table, it is observed that the optimized solution presented better results than the initial condition both in terms of rigidity and in terms of the first mode of vibration of the structure.

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TABLE III
SUMMARY OF RESULTS

Analyzes	Initial condition	Optimized proposal
Fist Frequency Mode	86,7Hz	136,4Hz
Second Frequency Mode	212,1Hz	529,4Hz
Stiffness	3,90N/mm	13,13N/mm

The evaluation of dynamic stiffness is presented at figure 14.

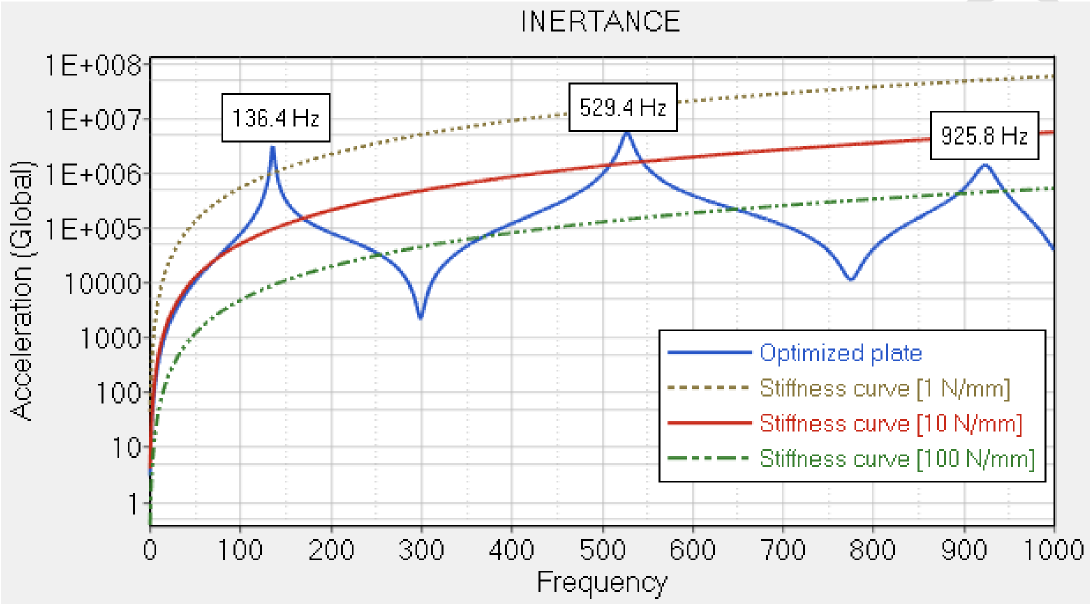

Fig. 14 Inertance Response with isocurve of dynamic stiffness of optimized plate

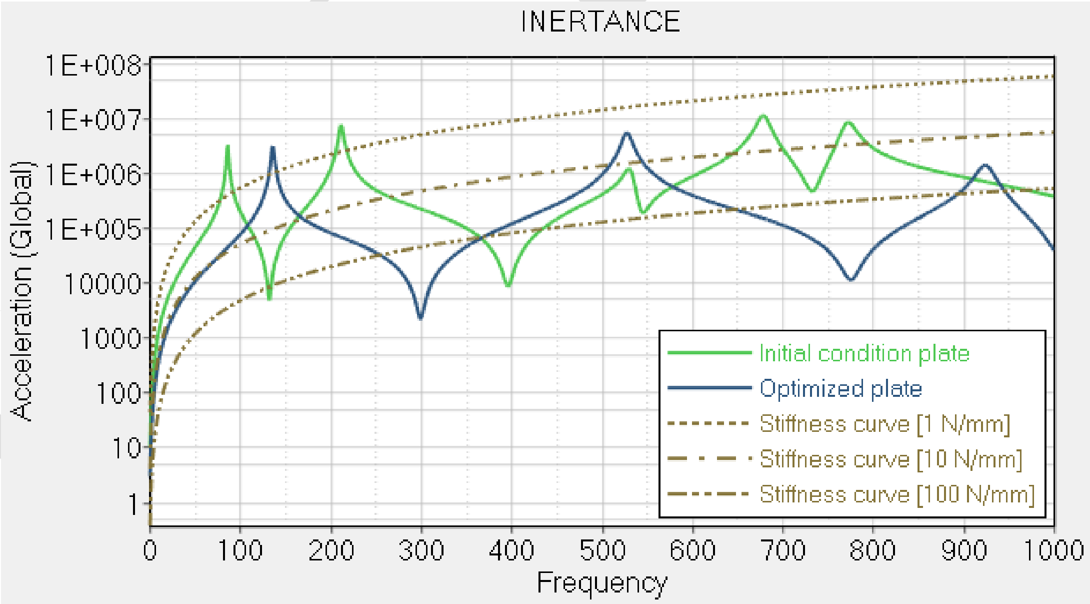

Fig. 15 Comparative initial condition and optimized plate

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IV. CONCLUSIONS

The comparison between the results obtained shows that the gain was significant for the optimized proposal in relation to the condition initially evaluated. This demonstrates that the use of topographic optimization allows good solutions for structural performance in the project analysis approach.

Through the Inertance analysis, it was possible to identify the critical points for a certain frequency range, an important condition in the design of projects where there is a forced excitation. It was also verified, that the control parameters in the topographic optimization has great influence for the final results obtained in the optimized structure, for example the angle and height, since a width is defined by the mesh refinement.

One of the ways that makes it easier to understand and evaluate the structure for a range of frequencies is the establishment of an objective function of dynamic stiffness instead of a simple static rigidity and of a minimum value for natural frequency.

The comparison presented in figure 15 allows to evaluate that the safe project region is given to higher values of dynamic stiffness.

Another important point is to find a solution through structural optimization in order, to obtain a reduction of the displacement amplitudes from the initial condition to optimized.

Already in the evaluation of accelerations, the amplitudes practically keep unaltered, but it happens at a frequency far superior to an initial condition, from 86,7Hz to 136,4Hz. Moreover, the evaluation of accelerations in the frequency domain allows us to easily perceive that the higher the stiffness the smaller the expected amplitude as evidenced by the graph of figure 15.

This study allowed the application of a calculation methodology in finite elements for a plate set in a simple and direct way and that can be used for models of greater complexity.

For example, to apply the strategy of improving the dynamic structural response of elements in the shape of plates of the vehicle body, especially those that suffer from the forced excitation in regions close to the engine, discharge pipe, etc.It will be easier to present solutions during the product development phase, since the dynamic curve allows a more efficient evaluation of the dynamic response of the structure.

As future work, it is expected to apply the methodology in the development of these elements in a vehicle as well as to extend to new design requirements in the field of noise.

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